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# Approximation of hydraulic turbine performance characteristics using optimal splines and neural networks

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**Abstract.** The performance characteristic of hydrounits is used very often in various control systems on hydro power plant for velocity control, power distribution, for displaying at different levels of automation pyramid: technicians, technologists, management and so on. Different systems do not have single unified description of the hydrounit's performance characteristic. It leads to growth of unnecessary calculations, errors and collisions. The study of the turbine efficiency can be automated by creating an appropriate mathematical model that approximates the initial performance characteristics of the turbine. This paper describes a method of approximation of the turbine performance characteristics using optimal two-dimensional cubic Hermit splines with free boundary conditions. The proposed method of hydrounit efficiency description allows to create single mathematical model and to use it in any control systems. The calculation load tests show high efficiency of the methods and confirm the possibility of their application in PLCs and servers.

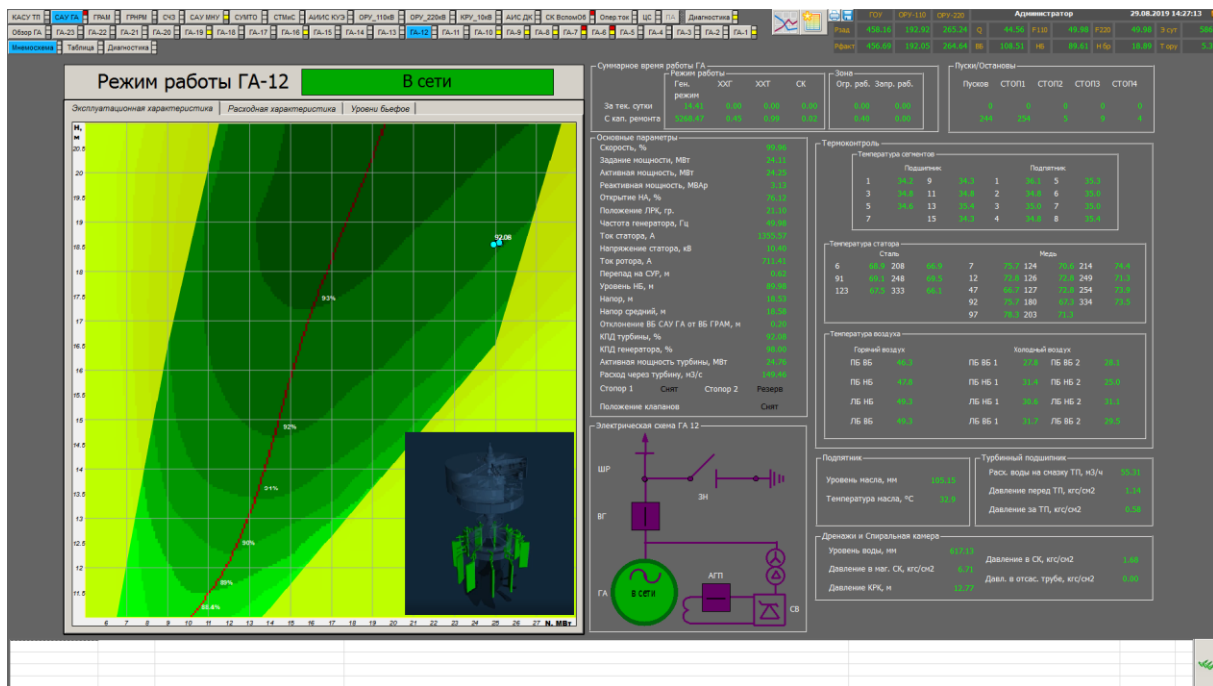
## 1. Introduction

According to modern trends of hydraulic power industry development it is supposed that main efforts should not be aimed at new HPPs building but at modernization and reconstruction of existing HPP equipment and at development of modern automation system to control these equipment [1].

The most important characteristics used in HPP automation system are the turbine performance ones. Usually the turbine performance characteristic is an efficiency coefficient as a function of the turbine power and head. They are used in different HPP systems: in the turbine speed governor, in active and reactive power group governor, in the system for rational control of hydrounits state [2,3] – to generate power limits, in the hydrounits automatic control system to calculate the hydrounit operation time in different modes and in restricted and forbidden operation zones, and for visualization at operator panels locally, dispatcher workstations and video walls [4]. Example from one of dispatcher system is shown on the figure 1.

In practice usually there is used table view of these characteristics [2,3,5–8], nowadays there is no unified representation of the hydrounit performance characteristics: even for one hydrounit different systems could use different representations for the same performance characteristic. Resulting from this it seems to be highly important to develop standardized approach for storage and calculation of the hydrounit performance characteristic, which could be used both for programmable logic controllers (speed governor, HPP active and reactive power group governor, hydrounit automatic control system), and for the server equipment (operator stations, servers and so on).





**Figure 1.** Hydrounit screen from dispatcher center.

At the same time, mathematic models of the hydrounit turbine performance characteristics could be significantly improved and simplified if they are approximated by continuous multivariable dependencies.

For this purpose there should be developed effective methods of complex function approximation differing with high processing speed and approximation high accuracy. For example, methods use optimal spline functions [9] or neural networks based on radial basis functions [10]. The first method has got high processing speed, the second one effectively adjusts to the approximated function specific.

Field of application of modern spline theory is quite wide. Spline functions are usually used for discrete functions interpolation problems or for smoothing of measurement accuracy in experimental dependencies. In particular, optimal parabolic splines were successfully used for approximation of reactive turbine-powered engines turbines and compressors characteristics.

Performance characteristics are usually developed on base of field tests. The dependencies developed have got complex specific view depending from a number of factors and, firstly, from the turbine type and power. Because of this, not every approximating function could be used for development of the hydrounit performance characteristic model. According to the research in spline-functions class the most suitable are cubic Hermit splines.

Cubic Hermit spline compares favorably with the other because at node points it has got two sets of conditions. Conditions of the spline passing through the node points are known from the task. What concerns setting of the first-order derivatives in the interpolation nodes, they could be used for the function graphic deformation in order to receive acceptable view of approximating dependency. Neural networks have got good ability to extrapolate the function where objective information is absent.

In this paper combined method of the hydrounit performance characteristic zooming (approximation) is offered. At the first stage the characteristic is approximated using radial basis neural network and the function is defined where it is not set. At the second stage using additional information there are used effective in processing speed optimal cubic Hermit splines [11]. Method of approximation of the hydrounit performance characteristic, described here, is different from traditional ones and, from our point of view, allows significantly increase quality of smooth approximation of the dependency generated.

## 2. Optimal cubic hermitian spline

Let the values of the function be given on the interpolation grid  $a = 0 < 1 < 2 < \dots < n = b$

$$y_0 = f(0), y_1 = f(1), \dots, y_n = f(n) \quad (1)$$

and the values of its derivatives

$$y'_0 = f'(0), y'_1 = f'(1), \dots, y'_n = f'(n). \quad (2)$$

The proposed integer interpolation grid greatly simplifies the derivation of formulas and does not affect the generality of the results obtained. From the integer grid by a linear transformation, it is easy to go to an arbitrary regular grid.

Consider the Hermitian cubic spline, in the following form:

$$S_{\mathfrak{g}}(x) = \begin{cases} y_0 + u_0 x + C_0 x^2 + D_0 x^3, & x \in (-\infty, 0]; \\ y_1 + u_1(x-1) + C_1(x-1)^2 + D_1(x-1)^3, & x \in (0, 1]; \\ \dots & \dots \\ y_{n-1} + u_{n-1}(x-n+1) + C_{n-1}(x-n+1)^2 + D_{n-1}(x-n+1)^3, & x \in (n-1, \infty]. \end{cases}$$

For the  $k$ -th interval of the Hermitian cubic spline

$$S_{\mathfrak{g}}^{(k)}(x) = y_k + u_k(x-k) + C_k(x-k)^2 + D_k(x-k)^3 \quad (3)$$

the first and second derivatives are:

$$\begin{aligned} \dot{S}_{\mathfrak{g}}^{(k)}(x) &= u_k + 2C_k(x-k) + 3D_k(x-k)^2, \\ \ddot{S}_{\mathfrak{g}}^{(k)}(x) &= 2C_k + 6D_k(x-k). \end{aligned}$$

For spline (3) is performed  $u_k = y'_k$ , therefore, in (2)  $y'_k$  can be replaced by  $u_k$ . Coefficients of the spline can be calculated from a system of linear equations.

$$\begin{cases} S_{\mathfrak{g}}^{(0)}(1) = y_0 + u_0 + C_0 + D_0 = y_1, \\ \dot{S}_{\mathfrak{g}}^{(0)}(1) = u_0 + 2C_0 + 3D_0 = u_1, \\ \\ S_{\mathfrak{g}}^{(k)}(k+1) = y_k + u_k + C_k + D_k = y_{k+1}, \\ \dot{S}_{\mathfrak{g}}^{(k)}(k+1) = u_k + 2C_k + 3D_k = u_{k+1}, \\ \\ S_{\mathfrak{g}}^{(n-1)}(n) = y_{n-1} + u_{n-1} + C_{n-1} + D_{n-1} = y_n, \\ \dot{S}_{\mathfrak{g}}^{(n-1)}(n) = u_{n-1} + 2C_{n-1} + 3D_{n-1} = u_n. \end{cases} \quad (4)$$

We are introduced auxiliary variables

$$\begin{aligned} d_k &= 3(y_{k+1} - y_k), \\ e_k &= -2(y_{k+1} - y_k), \end{aligned} \quad (5)$$

then from the system of equations, taking into account (5), one can find the coefficients of the Hermitian cubic spline (3):

$$\begin{cases} C_k = d_k - u_{k+1} - 2u_k, \\ D_k = e_k + u_{k+1} + u_k. \end{cases} \quad (6)$$

Recall here  $u_k = y'_k$ .

We practically generated interpolating cubic Hermit spline, but in case of real tasks solving, with rare exceptions, we don't know the conditions (2). They are unknown for the hydrounit performance characteristic. From the other hand, we can say that coefficients  $u_0, u_1, \dots, u_n$  are free changing variables and we could find their values by solving optimization task. For explicitly set functions as this criterion spline mean-root-square curvature estimation criterion could be used:

$$J = \int_0^n \ddot{S}_s^2(x) dx \rightarrow \min_{u_0, \dots, u_n} . \quad (7)$$

or

$$J = \int_0^n \ddot{S}_s^2(x) dx = \sum_{k=0}^{n-1} \int_k^{k+1} (\ddot{S}^{(k)}(x))^2 dx = \sum_{k=0}^{n-1} J_k \rightarrow \min_{u_0, \dots, u_n} . \quad (8)$$

Converting the task of generation of cubic Hermit spline into the optimizing task simplifies solving of the approximation task for experimental dependencies, but results in additional difficulties. Direct methods of multidimensional task of global optimization (GO) [12] are referred to NP class of complex tasks [13] and are rather complex.

Criterion  $J$  is an analog of flexible rail bending energy estimation, calculated on base of set of cross-sections passing through the network nodes. At the same time it provides minimum spline mean-root-square curvature what limits spline oscillation between interpolation nodes.

### 2.1. Optimal cubic Hermit spline generation

Let us consider subtest  $J_k$ .

$$\begin{aligned} J_k &= \int_k^{k+1} (\ddot{S}_s^{(k)}(x))^2 dx = \int_k^{k+1} (2C_k + 6D_k(x - x_k))^2 dx = \\ &= 4C_k^2 + 6C_k D_k + 12D_k^2 . \end{aligned} \quad (9)$$

Expression (9) in details using (5-6) seems as follows:

$$\begin{aligned} J_k &= 10u_{k+1}^2 + (18e_k - 2d_k + 22u_k)u_{k+1} + (e_k + d_k)(6d_k - 12u_k) + \\ &+ 12(e_k + 2u_k) + 4(d_k - 2u_k)^2 . \end{aligned}$$

Considering that we are not interested in optimal value of the subtest  $J_k$ , but only in optimal values of optimized variables, accept criterion  $\tilde{J}_k = \frac{1}{2} J_k$ , then

$$\begin{aligned} \tilde{J}_k &= 5u_{k+1}^2 + 11u_k u_{k+1} + 8u_k^2 + (9e_k - d_k)u_{k+1} + (6e_k - 5d_k)u_k + \\ &+ 2d_k^2 + 3d_k e_k + 6e_k . \end{aligned} \quad (10)$$

If accept designations

$$\begin{aligned} w_1^{(k)} &= 5; w_2^{(k)} = 11; w_3^{(k)} = 8; w_4^{(k)} = 9e_k - d_k; w_5^{(k)} = 6e_k - 5d_k; \\ w_6^{(k)} &= 2d_k + 3d_k e_k + 6e_k , \end{aligned} \quad (11)$$

subtests  $\tilde{J}_k$  for variables  $u_0, u_1, \dots, u_{n-1}$  make up quadric form

$$\tilde{J}_k = w_1^{(k)} u_{k+1}^2 + w_2^{(k)} u_k u_{k+1} + w_3^{(k)} u_k^2 + w_4^{(k)} u_{k+1} + w_5^{(k)} u_k + w_6^{(k)} , \quad (12)$$

and criterion (8) represents a sum of quadric forms of the view (12). Value of every effectiveness subtest  $J_k(y_k, y_{k+1}, u_k, u_{k+1})$  depends from the spline behavior at the previous interval of interpolation grid (1), from conditions of smooth conjunction of cubic parabolas and controls  $u_{k+1}$ , chosen at each interval of interpolation. It is considered that control  $u_k$  is known from the previous interval. As a result, there is a possibility to replace task of GO criterion (8) with the task of dynamic programming, executing choice of optimal controls depending on the spline behavior at node points.

Procedure of generation of optimal control using dynamic programming method is divided into two stages: preliminary and final.

At the preliminary stage optimal strategy of the spline generation is developed. The process starts from the last interval of the interpolating grid when at assumption that up to the point  $(n-1, y_{n-1})$  spline is optimal the task of choice of optimal control  $u_n$  is solved:  $\min_{u_n} J_{n-1}(u_n)$ . In this case for the interval

(n-1; n] and criterion  $\tilde{J}_{n-1} = w_1^{(n-1)} u_n^2 + w_2^{(n-1)} u_{n-1} u_n + w_3^{(n-1)} u_{n-1}^2 + w_4^{(n-1)} u_n + w_5^{(n-1)} u_{n-1} + w_6^{(n-1)}$ , optimal strategy will be:

$$u_n^{opt} = -\frac{w_2^{(n-1)} u_{n-1} + w_4^{(n-1)}}{2w_1^{(n-1)}}. \quad (13)$$

At the interval (n-2; n] optimization is executed on base of criterion

$$W_{n-2}(u_{n-2}, u_{n-1}) = \tilde{J}_{n-2}(u_{n-2}, u_{n-1}) + \tilde{J}_{n-1}(u_{n-1}, u_n^{opt}) \rightarrow \min_{u_{n-1}}. \quad (14)$$

Quadric form coefficients (14) with regard to optimal control  $u_n^{opt}$  could be calculated as follows:

$$\begin{aligned} \tilde{w}_1^{(n-2)} &= \frac{4w_1^{(n-1)}w_3^{(n-1)} - (w_2^{(n-1)})^2}{4w_1^{(n-1)}}, \\ \tilde{w}_4^{(n-2)} &= \frac{2w_1^{(n-1)}w_3^{(n-1)} - w_2^{(n-1)}w_1^{(n-1)}}{4w_1^{(n-1)}}, \\ \tilde{w}_6^{(n-2)} &= \frac{4w_1^{(n-1)}w_6^{(n-1)} - (w_4^{(n-1)})^2}{4w_1^{(n-1)}}. \end{aligned} \quad (15)$$

Summing coefficients (15) with the quadric form coefficients  $\tilde{J}_{n-2}(u_{n-1}, u_n)$ , we get formulas to calculate quadric form coefficients of the criterion  $W_{n-2}(u_{n-2}, u_{n-1})$ :

$$\begin{cases} \tilde{w}_1^{(n-2)} = w_1^{(n-2)} + \tilde{w}_1^{(n-2)}, \\ \tilde{w}_2^{(n-2)} = w_2^{(n-2)}, \\ \tilde{w}_3^{(n-2)} = w_3^{(n-2)}, \\ \tilde{w}_4^{(n-2)} = w_4^{(n-2)} + \tilde{w}_4^{(n-2)}, \\ \tilde{w}_5^{(n-2)} = w_5^{(n-2)}, \\ \tilde{w}_6^{(n-2)} = w_6^{(n-2)} + \tilde{w}_6^{(n-2)}. \end{cases} \quad (16)$$

Minimizing criterion (14) according to control  $u_{n-1}$ , we can find optimal strategy  $u_{n-1}^{opt}$ .

The process is continuing until we reach the first interval (0; 1]. In this case for the criterion

$$W_0(u_0, u_1) = \tilde{J}_0(u_0, u_1) + \tilde{J}_1(u_1, u_2^{opt}) + \dots + \tilde{J}_{n-1}(u_{n-1}, u_n^{opt})$$

we can find optimal controls  $u_0, u_1$  using formulas

$$\begin{cases} u_0^{opt} = -\frac{2\tilde{w}_2^{(1)}\tilde{w}_5^{(1)} + \tilde{w}_2^{(1)}\tilde{w}_4^{(1)}}{4\tilde{w}_1^{(1)}\tilde{w}_3^{(1)} - (\tilde{w}_2^{(1)})^2}, \\ u_1^{opt} = \frac{\tilde{w}_2^{(1)}\tilde{w}_5^{(1)} - 2\tilde{w}_3^{(1)}\tilde{w}_4^{(1)}}{4\tilde{w}_1^{(1)}\tilde{w}_3^{(1)} - (\tilde{w}_2^{(1)})^2}. \end{cases} \quad (17)$$

As a result we have got analytical expressions to define all optimal strategies  $u_0^{opt}, u_1^{opt}, \dots, u_n^{opt}$ , i.e. missing values of approximation derivatives at spline approximation in the spline node points (2). Applying found values of variables  $u_0^{opt}, u_1^{opt}, \dots, u_n^{opt}$  to the expression (2), we generate optimal cubic Hermit spline.

## 2.2. Multidimensional dependenced approximation by optimal cubic Hermit spline

For approximation of two-dimensional dependencies one-dimension optimal cubic Hermit spline, described above, is used.

Let us set interpolating grid for two-dimension function  $z = f(x, y)$ :

$$\begin{cases} \Delta_x : 0, 1, \dots, n; \\ \Delta_y : 0, 1, \dots, n, \end{cases} \quad (18)$$

and at interpolating grid nodes the values of the function  $f_{ij}$  are known.

Two-dimensions interpolating optimal cubic spline is generated according to well-known scheme [11].

1. Let us generate  $n+1$  optimal cubic spline  $S_k(x|y_k)$ ,  $y_k \in \{0, 1, \dots, n\}$  for the set values of the variable  $y_k$ .

2. To calculate spline-function in the point  $(x^*, y^*)$  let us calculate  $n+1$  value of splines  $\tilde{z}_k = S_k(x^*|y_k)$  in the point  $x = x^*$ . We will get one more task of the function interpolation at the grid  $\Delta_y : 0, 1, \dots, n$  with the function values at the grid nodes  $\tilde{z}_0, \tilde{z}_1, \dots, \tilde{z}_n$ .

3. Let us generate one more optimal cubic spline  $S_y(x^*, y)$ . Its calculation in the point  $y = y^*$  will get us target value  $z^* = S_y(x^*, y^*)$ .

Now, knowing two-dimensions optimal cubic Hermit spline calculation method, let's generate with its help approximation scheme for two-dimensions experimental dependencies.

In the grid (18) accept matrix of changing variables — function values in the interpolation nodes:

$$Z = \begin{vmatrix} z_{00} & z_{01} & \dots & z_{0n} \\ z_{10} & z_{11} & \dots & z_{1n} \\ \dots & \dots & \dots & \dots \\ z_{n0} & z_{n1} & \dots & z_{nn} \end{vmatrix} \quad (19)$$

Assume at randomized grid there is set the hydroturbine performance characteristic got upon the results of the field tests

$$F = \begin{vmatrix} x_1 & y_1 & f_1 \\ x_2 & y_2 & f_2 \\ \dots & \dots & \dots \\ x_M & y_M & f_m \end{vmatrix} \quad (20)$$

Using ordinary least squares technique (OLS) consider optimization task for mean-root-square function

$$I = \sum_{k=1}^M (S_{\mathcal{O}}(x_k, y_k) - f_k)^2 \xrightarrow{z_g \in Z} \min. \quad (21)$$

As a result rational combinations of reference values of the matrix (19)  $Z^*$ , minimizing mean-root-square accuracy of source function approximation will be sorted out.

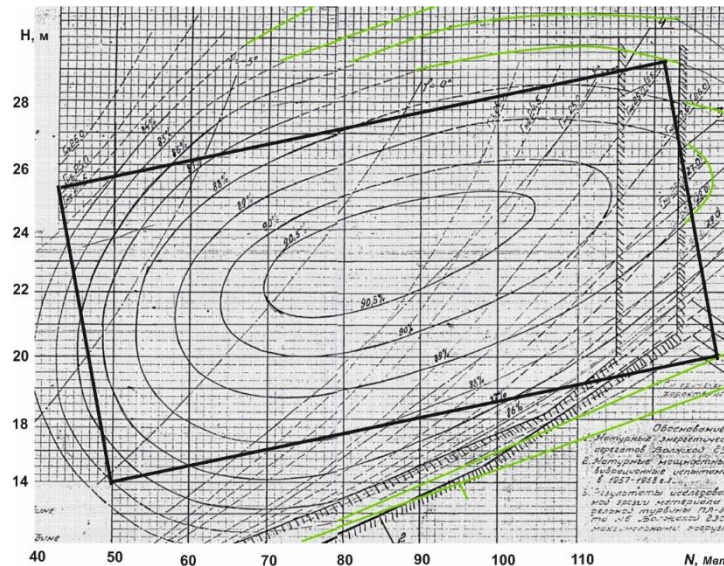
### 3. Approximation of multidimensions dependencies with combined method using radial basis neural networks

In the recent times in the function approximation tasks neural networks on base of radial basis functions [10] are finding wider and wider application. A significant scope of literature is devoted to this topic. Neural networks — powerful modeling method making it possible to simulate extremely complex dependencies, and the networks based on RBF compare favorable with easy usage. The user should have only the set of heuristic knowledge required for preparation of the source data.

As an example let's consider the approximation task for Volzjskaya HPP пп-586-ББ-930 hydroturbine characteristic (dependency of efficiency rate from power and head  $\eta_T(N, H)$ ). From the figure 2 we can see that part of the source variables space has no any information. This circumstance causes difficulties during the function approximation both for neural network and for the spline because requires solving of the complex task of the function extrapolation outside the limits of working area of

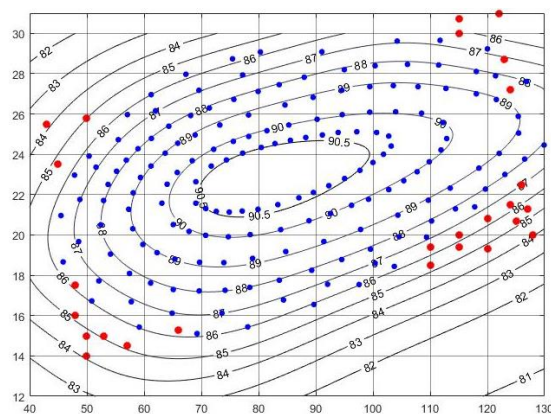


the function graphics. But working area of the function approximated with the help of linear representation could be inscribed more tightly for example, into the parallelogram, as it is shown in the picture.

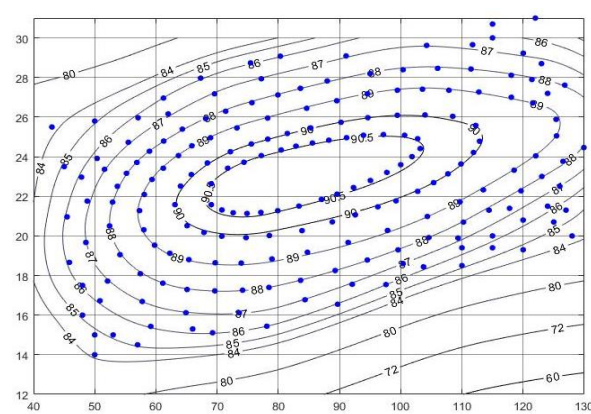


**Figure 2.** Hydrounit ПЛ-586-ББ-930 performance characteristic.

Based on RBF neural networks easily cope with the approximation task for the HU performance characteristics. In the figure 3 there are shown results of the approximation executed with the help of Matlab2016b system package at the following parameters: permissible mean-root-square error (MRSE) GOAL=0.42 and spread SPREAD=1.5. As a result, there was reached quite high accuracy of the approximation inside the characteristic working area and acceptable accuracy of the function extrapolation outside this area. Reached MRSE was 0.1193, neuron number in the hidden layer – 15.



**Figure 3.** Results of ПЛ-586-ББ-930 HU characteristic approximation with RBF neuron network ● – results of the HU bench tests, ● – extrapolation of the HU characteristic with the neural network.



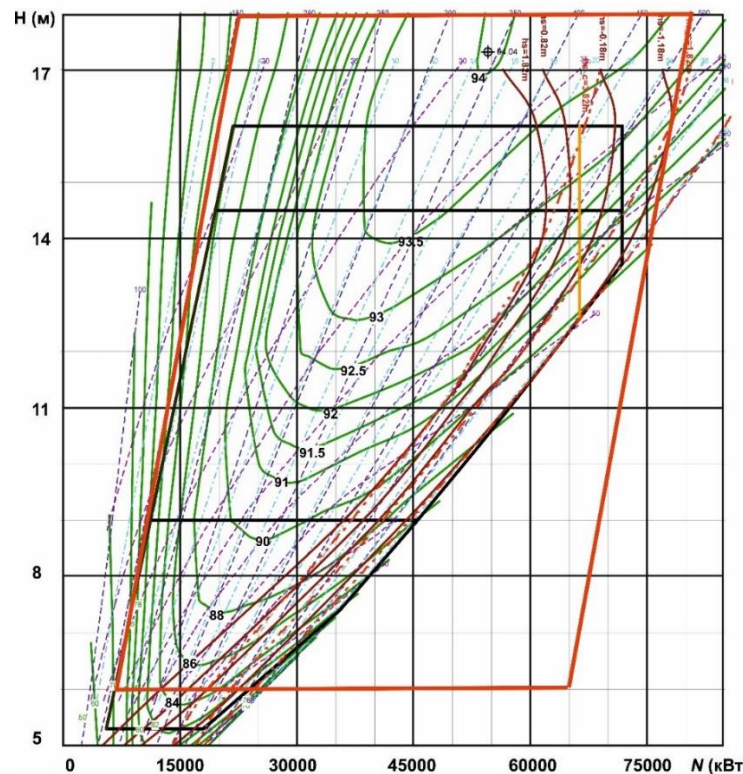
**Figure 4.** Results of ПЛ-586-ББ-930 HU characteristic approximation with optimal cubic Hermit spline.

Approximation of the same characteristic completed with approximation nodes at the parallelogram “blank” spaces, calculated with the help of the system neural model using optimal Hermit spline allows significantly improves the HU performance characteristic approximation in its operating area (see figure 4, MRSE=0.0757).



Essential value in this case has the fact that average speed of the program for HU characteristic calculation in the point (calculation of the function  $\eta_T(N, H)$ ) for spline is 67 times more than for neural network. Analog indexes concerning spline-approximation speed were reached and for the other characteristics. In fact, spline-functions operate in two orders faster than neural networks models.

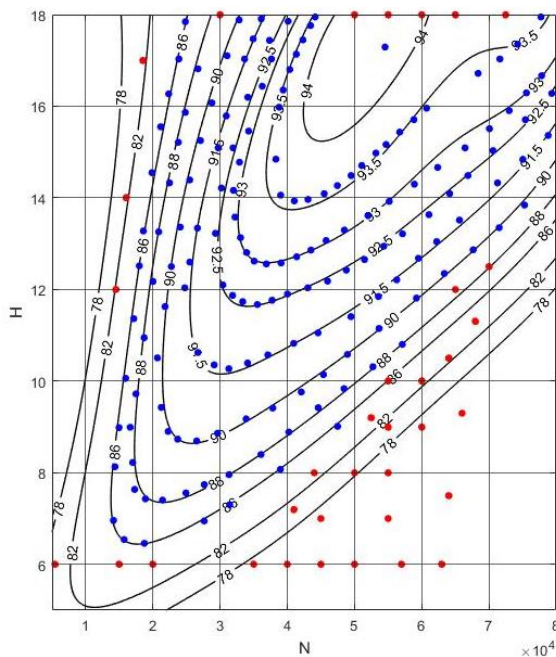
Let's take a look at the another example with approximation for hydrounit of Uglichskaya HPP (figure 5).



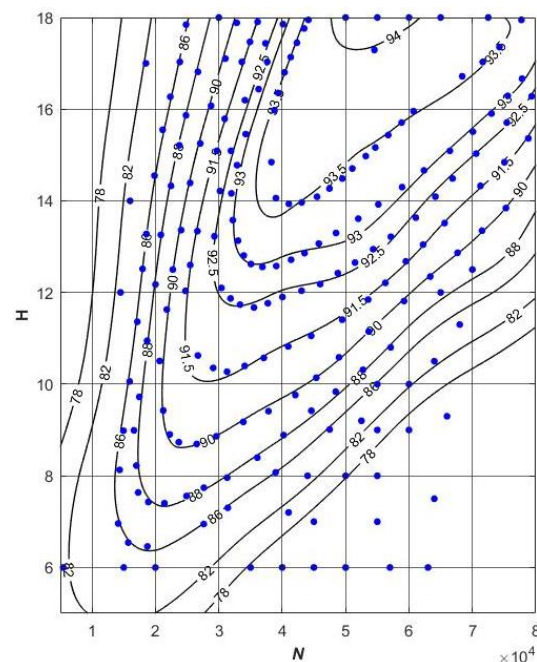
**Figure 5.** Performance characteristic of hydrounit of Uglichskaya HPP.

The peculiarity of the unit performance characteristic is triangular form that significantly reduces valuable part of function in the space of independent variables  $N$ - $H$ . To decrease uncertainty of approximation valuable part of function is placed to parallelogram shown on the picture with red lines. The combined graphic and analytical method of approximation used in the automation hydro power plants as de facto standard faces problem. The methodical error is affected by errors of measurements (in the HPP or in the laboratory), inaccuracy of mathematical models calculation of main parameters and at last human factor when graphically drawing the corresponding contours of the formed functional dependencies. The performance characteristic of Uglichskaya HPP's HU obviously contains all mentioned defects. As shown on the Figure 5. lines for equal performance coefficient are especially poorly drawn. In the case the approximation task of could be replaced by task of reconstruction of functional dependencies  $\eta_T(N, H)$  roughly represented on figure 5 the reconstruction task was solved on base of RBF neural networks.

On the figure 6 there are shown approximation results provided with NNT package in Matlab2016b at the following parameters: permissible mean-root-square error (MRSE) GOAL=0.36 and spread SPREAD=3.1. High accuracy of the approximation inside HU's working area was reached and the result function behaves appropriately in extrapolation area. Reached MRSE was 0.1549, neuron number in the hidden layer – 20.



**Figure 6.** Results of RBF neural network approximation for HU of Uglichskaya HPP characteristic: ● - results of experiments on real HU, ● - extrapolation of HU characteristic with neural network.



**Figure 7.** Approximation results for characteristic of HU of Uglichskaya HPP with optimal cubic Hermit splines.

Taking into account the quality of source information the calculated result is quite acceptable. The received neural network approximation could be used in automation systems, nevertheless experiments showed calculation speed of neuro network approximation worse by 73 times than optimal cubic Hermit splines. Considering additional nodes in extrapolation area (red dots on the figure 6) approximation was made based on optimal cubic Hermit splines. It allows significantly improve approximation of HU's performance characteristic in its working area (figure 7, MRSE = 0.1371). Estimations of calculation speed shows that spline-functions  $\eta_T(N, H)$  are almost 2 orders of magnitude faster than neural networks models.

#### 4. Conclusion

The article offers a new method of the hydroturbine performance characteristics approximation. Complexity of the task is considered in need for combination of reaching acceptable accuracy of approximation of nonsmooth, specific performance characteristics of the HU with high speed of calculating programs intended for programmable logic controllers. Offered combined method for approximation of the hydrounit performance characteristic based on application of neural networks RBF at the stage of approximation and optimal cubic Hermit splines at the stage of application of the dependencies generated in the whole scope solves the task set.

Application of neural networks technologies is caused by their extraordinary ability for qualitative extrapolation of the approximated dependency behavior in the approximate areas where the information is objectively absent. Extension of definition of the appropriate information makes it possible to use optimal cubic Hermit splines for solving of the end task – the hydrounit performance characteristic approximation. In the paper original method for calculation of optimal cubic spline coefficients is represented. Cubic Hermit splines, being continuous and smooth functions in the first-order derivative have got additional degrees of freedom in the interpolation nodes and can more precisely “fit in” the specific of complex functions having significant variations of the function gradient in separate places.

From the other hand, splines, being functions “composed” from cubic parabolas pieces have computational complexity comparable to the third order polynomial.

Simulation experiments have shown high effectiveness of optimal cubic Hermit splines in solving the task of approximation of the hydrounit performance characteristics as well as confirmed possibility of their application both in PLCs and at the server equipment.

Application of the approach offered makes it possible for every HPP unit generate single unified model of the performance characteristic, which can be used by all systems where it is required, which results in elimination of recalculations, errors and collisions in different systems.

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